





# **Student Edition**

**Stephen Hake** 



A Harcourt Achieve Imprint

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– Stephen Hake

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#### **ABOUT THE AUTHOR**

Stephen Hake has authored six books in the **Saxon Math** series. He writes from 17 years of classroom experience as a teacher in grades 5 through 12 and as a math specialist in El Monte, California. As a math coach, his students won honors and recognition in local, regional, and statewide competitions.

Stephen has been writing math curriculum since 1975 and for Saxon since 1985. He has also authored several math contests including Los Angeles County's first Math Field Day contest. Stephen contributed to the 1999 National Academy of Science publication on the Nature and Teaching of Algebra in the Middle Grades.

Stephen is a member of the National Council of Teachers of Mathematics and the California Mathematics Council. He earned his BA from United States International University and his MA from Chapman College.

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Strands Key: NO = Number and Operations A = Algebra G = Geometry M = Measurement DAP = Data Analysis and Probability PS = Problem Solving CM = Communication RP = Reasoning and Proof C = Connections R = Representation

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### LETTER FROM THE AUTHOR



### Dear Student,

We study mathematics because it plays a very important role in our lives. Our school schedule, our trip to the store, the preparation of our meals, and many of the games we play involve mathematics. The word problems in this book are often drawn from everyday experiences.

When you become an adult, mathematics will become even more important. In fact, your future may depend on the mathematics you are learning now. This book will help you to learn mathematics and to learn it well. As you complete each lesson, you will see that similar problems are presented again and again. *Solving each problem day after day is the secret to success.* 

Your book includes daily lessons and investigations. Each lesson has three parts.

- The first part is a Power Up that includes practice of basic facts and mental math. These exercises improve your speed, accuracy, and ability to do math *in your head*. The Power Up also includes a problem-solving exercise to help you learn the strategies for solving complicated problems.
- 2. The second part of the lesson is the New Concept. This section introduces a new mathematical concept and presents examples that use the concept. The Lesson Practice provides a chance for you to solve problems using the new concept. The problems are lettered a, b, c, and so on.
- **3.** The final part of the lesson is the Written Practice. This section reviews previously taught concepts and prepares you for concepts that will be taught in later lessons. Solving these problems will help you practice your skills and remember concepts you have learned.

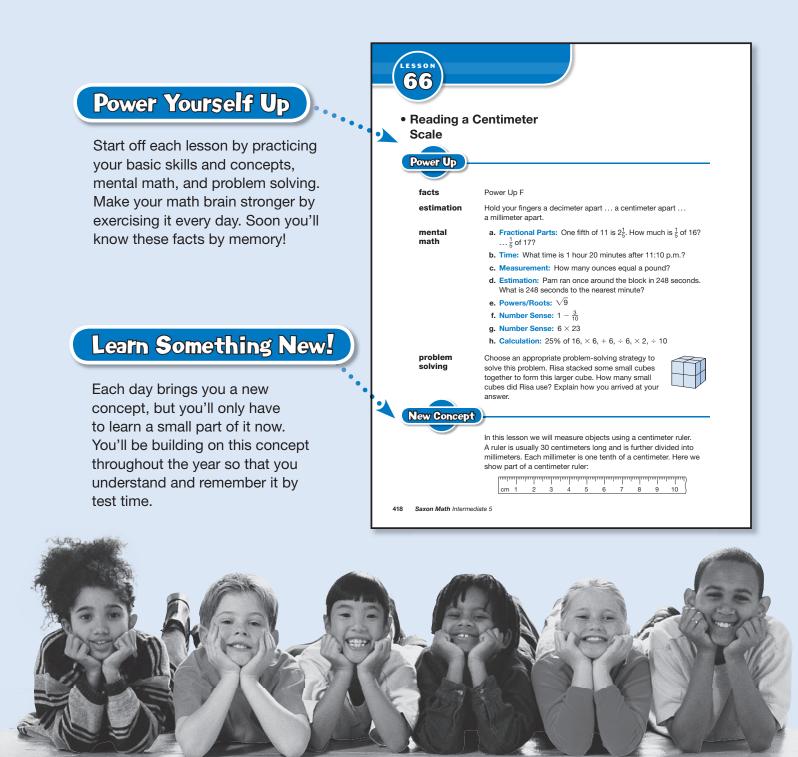
Investigations are variations of the daily lesson. The investigations in this book often involve activities that fill an entire class period. Investigations contain their own set of questions but do not include Lesson Practice or Written Practice.

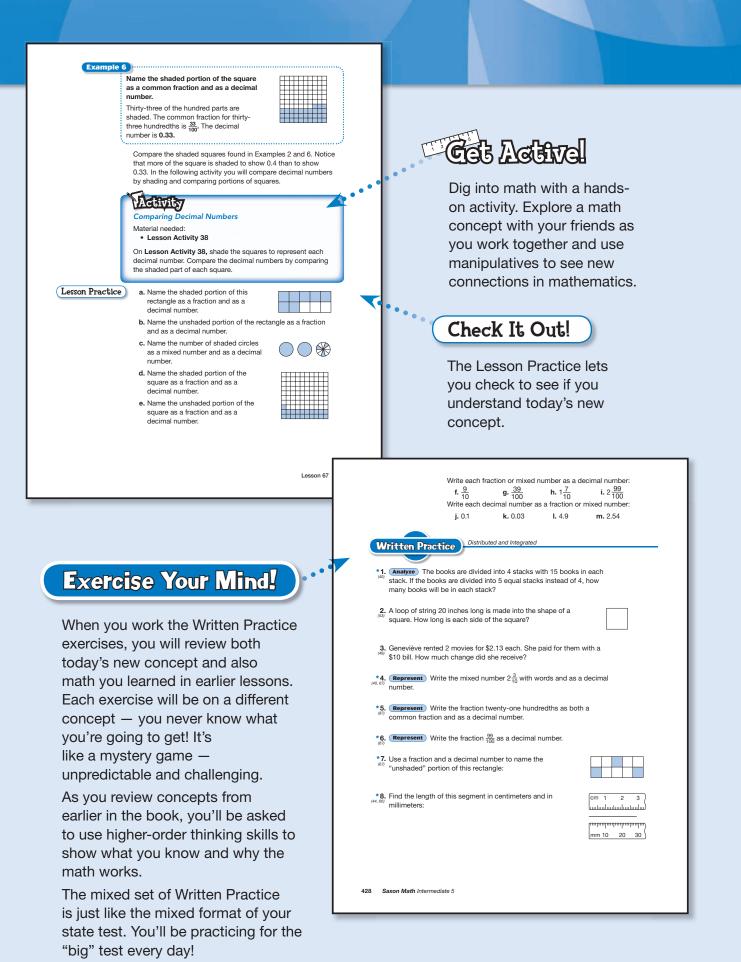
Remember to solve every problem in each Lesson Practice, Written Practice, and Investigation. Do your best work, and you will experience success and true learning that will stay with you and serve you well in the future.

Temple City, California

### HOW TO USE YOUR TEXTBOOK

**Saxon Math Intermediate 5** is unlike any math book you have used! It doesn't have colorful photos to distract you from learning. The Saxon approach lets you see the beauty and structure within math itself. You will understand more mathematics, become more confident in doing math, and will be well prepared when you take high school math classes.





## HOW TO USE YOUR TEXTBOOK

### Become an Investigator!

Dive into math concepts and explore the depths of math connections in the Investigations.

Continue to develop your mathematical thinking through applications, activities, and extensions.



#### Focus on

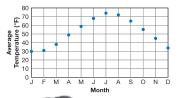
#### • Line Graphs

Often we are interested in seeing the changes in data that occur over a period of time. Below we show the average temperature in the city of Boston for each month of the year.

Average Boston Temperature									
Month	Temp.		Month	Temp.					
January	30°F		July	74°F					
February	31°F		August	72°F					
March	38°F		September	65°F					
April	49°F		October	55°F					
May	59°F		November	45°F					
June	68°F		December	34°F					

The temperature is lowest in January and February. Then the weather warms up steadily until summer arrives. It stays warm through August and then cools steadily after that. In December the temperature is almost as low as it is at the beginning of the year.

To show the change of temperature over time, we can use a line graph. We will draw the **line graph** on a grid. First we label each of the 12 months along the grid's **horizontal axis**. Then we label temperatures from  $^{\circ}$ F through 80°F along the grid's **vertical axis**. We label up to 80° on the grid because we need to graph temperatures as high as 74°. We choose our interval to be 10°F on the vertical axis. We could use a smaller interval instead (such as 5°), but then our grid would be bigger. Above each month, we place a dot at a height equal to the normal temperature for that month.



Investigation 6 383

### Focus on

### Problem Solving

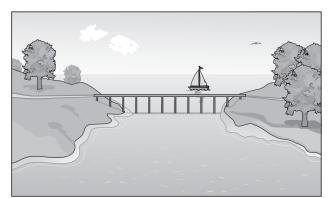
We study mathematics to learn how to use tools that help us solve problems. We face mathematical problems in our daily lives, in our work, and in our efforts to advance our technological society. We can become powerful problem solvers by improving our ability to use the tools we store in our minds. In this book we will practice solving problems every day.

This lesson has three parts:

Problem-Solving Process The four steps we follow when solving problems.Problem-Solving Strategies Some strategies that can help us solve problems.Writing and Problem Solving Describing how we solved a problem or formulating a problem.

#### Four-Step Problem-Solving Process

Solving a problem is like arriving at a destination, so the process of solving a problem is similar to the process of taking a trip. Suppose we are on the mainland and want to reach a nearby island.



Step	Problem-Solving Process	Taking a Trip				
1	<b>Understand</b> Know where you are and where you want to go.	We are on the mainland and want to go to the island.				
2	Plan your route.	We might use the bridge, the boat, or swim.				
3	<b>Solve</b> Follow the plan.	Take the journey to the island.				
4	<b>Check</b> Check that you have reached the right place.	Verify that you have reached your desired destination.				

When we solve a problem, it helps to ask ourselves some questions along the way.

Step	Follow the Process	Ask Yourself Questions
1	Understand	What information am I given? What am I asked to find or do?
2	Plan	How can I use the given information to solve the problem? What strategy can I use to solve the problem?
3	Solve	Am I following the plan? Is my math correct?
4	Check	Does my solution answer the question that was asked? Is my answer reasonable?

Below we show how we follow these steps to solve a word problem.

Example	1
	Carmen, Destiny, and Sergio each had 25 square tiles. Carmen arranged her tiles to make one square. Destiny arranged her 25 tiles to make two squares. How many tiles were in each of Destiny's squares? Can Sergio make three squares using all 25 tiles?Image: Can Sergio imake three squares using all 25 tiles?
	Step 1: Understand the problem. We are
	given the following information:
	Each person had 25 tiles.
	<ul> <li>Carmen made one square, and it is shown.</li> </ul>
	<ul> <li>Destiny made two squares, not shown.</li> </ul>
	We are asked for the number of tiles in each of Destiny's squares and if it is possible for Sergio to make three squares with 25 tiles.
	<b>Step 2: Make a plan.</b> We cannot find the number of tiles in Destiny's squares by adding, subtracting, dividing, or multiplying 25 and 2. If we have 25 tiles, we can try making two squares with them. We can also draw pictures of squares, which is what we will do.
	<b>Step 3: Solve the problem.</b> We draw pictures of some of the squares we can make with square tiles.
	Image: 1 tile     Imag

2

We see that we can make squares with 1, 4, 9, and 16 tiles. We notice that 9 + 16 equals 25, so we find that Destiny's two squares had 9 tiles and 16 tiles. We also see that no combination of three squares totals 25 tiles, so Sergio cannot make three squares using all 25 tiles.

**Step 4: Check your answer.** We look back at the problem to see if we have used the correct information and have answered the question. By drawing a picture, we found the two squares Destiny made using 25 square tiles. We also found that it is not possible for Sergio to make three squares with 25 tiles. By checking the drawing of each square, we find that our answer is reasonable.

#### Example 2

Ms. Jones used a paper cutter to cut pieces of construction paper for the students in her class. She cut the sheet in half, placed one half on top of the other, and then made a second cut that cut both pieces in half. If she continues this process, how many cuts will she need to make in order to have one small piece of construction paper for each of her 30 students?

**Step 1: Understand the problem.** Ms. Jones is cutting a sheet of construction paper so that each of her 30 students will have one small piece of paper. First she cuts one sheet, making two. Then she cuts two sheets, making four. She continues the process until she has enough pieces of construction paper.

**Step 2: Make a plan.** First she has one piece, then two pieces, then four pieces. We see that there is a pattern. We will continue the pattern and make a list.

**Step 3: Solve the problem.** We make a list that shows the number of pieces after each cut.

Cuts	(Uncut)	First	Second	Third	Fourth	Fifth
Number of Pieces	1	2	4	8	16	32

Each cut doubles the number of pieces. We find that after the fifth cut there are 32 pieces, enough for each student to have one.

**Step 4: Check your answer.** We look back at the problem to be sure we have used the correct information and have answered the question. We know that our answer is reasonable because cutting each stack in half doubled the number of pieces after each cut. There were not enough pieces after four cuts, but after the fifth cut, there were more than enough pieces of paper.

- 1. List in order the four steps in the problem-solving process.
- 2. What two questions do we answer to understand a problem?

Refer to the following problem to answer questions 3–8:

Mrs. Rojas is planning to take her daughter, Lena, and her friend, Natalie, to see a movie. The movie starts at 5:00 p.m. She wants to arrive at the theater 20 minutes before the movie starts. It will take 15 minutes to drive to Natalie's house. It is 10 minutes from Natalie's house to the theater. At what time should Mrs. Rojas leave her house?

- 3. **Connect** What information are we given?
- 4. **Verify** What are you asked to find?
- 5. Which step of the four-step problem-solving process did you complete when you answered problems 3 and 4?
- 6. Describe your plan for solving the problem.
- 7. **Explain** Solve the problem by following your plan. Show your work. Write your solution to the problem in a way someone else will understand.
- 8. Check your work and your answer. Look back to the problem. Be sure you use the information correctly. Be sure you found what you were asked to find. Is your answer reasonable?

#### **Problem-Solving Strategies**

As we consider how to solve a problem, we choose one or more strategies that seem to be helpful. Referring to the picture at the beginning of this lesson, we might choose to swim, to take the boat, or to cross the bridge to travel from the mainland to the island. Other strategies might not be as effective for the illustrated problem. For example, choosing to walk or bike across the water are strategies that are not reasonable for this situation.

When solving mathematical problems, we also select strategies that are appropriate for the problem. **Problem-solving strategies** are types of plans we can use to solve problems. Listed below are ten strategies we will practice in this book. You may refer to these descriptions as you solve problems throughout the year.

Act it out or make a model. Moving objects or people can help us visualize the problem and lead us to the solution.

**Use logical reasoning.** All problems require reasoning, but for some problems we use given information to eliminate choices so that we can more easily find the solution. Usually a chart, diagram, or picture can be used to organize the given information and to make the solution more apparent.

**Draw a picture or diagram.** Sketching a picture or a diagram can help us understand and solve problems—especially problems about graphs, maps, or shapes.

Write a number sentence or equation. Fitting the given numbers into equations or number sentences, and then finding the unknown numbers, can help us solve many word problems.

Make it simpler. Using smaller numbers or fewer items can make some complicated problems easier. Solving the simpler problem might allow us to see a pattern or method that can help us solve the complex problem.

**Find/Extend a pattern.** Identifying a pattern that helps us predict what will come next as the pattern continues might lead to the solution.

Make an organized list. Making a list can help us organize our thinking about a problem.

**Guess and check.** Guessing the answer and trying the guess in the problem might start a process that leads to the answer. If the guess is not correct, use the information from the guess to make a better guess. Continue to improve your guesses until you find the answer.

Make or use a table, chart, or graph. Arranging information in a table, chart, or graph can help us organize and keep track of data. This might reveal patterns or relationships that can help us solve the problem.

**Work backwards.** Finding a route through a maze is often easier by beginning at the end and tracing a path back to the start. Likewise, some problems are easier to solve by working back from information that is given toward the end of the problem to information that is unknown near the beginning of the problem.

9. Name some strategies used in this lesson.

The chart below shows where each strategy is first introduced in this textbook.

Strategy	Lesson
Act It Out or Make a Model	12
Use Logical Reasoning	3
Draw a Picture or Diagram	21
Write a Number Sentence or Equation	17
Make It Simpler	58
Find/Extend a Pattern	1
Make an Organized List	4
Guess and Check	18
Make or Use a Table, Chart, or Graph	40
Work Backwards	13

#### Writing and Problem Solving

Sometimes a problem will ask us to explain our thinking. This helps us measure our understanding of math and is easy to do.

- Explain how you solved the problem.
- Explain how you know your answer is correct.
- Explain why your answer is reasonable.

For these situations, we can describe the way we followed our plan. This is a description of the way we solved Example 1.

We drew pictures to find the number of tiles needed to make squares of different sizes. Then we found two squares that totaled 25 tiles. We also looked for three squares that totaled 25 tiles.

**10.** Write a description of how we solved the problem in Example 2.

Other times we will be asked to write a problem for a given equation. Be sure to include the correct numbers and operations to represent the equation.

**11.** Write a word problem for 9 + 16 = 25.

LESSON 1	
Sequences	
• Digits	
Power Up	
facts	Power Up A <sup>1</sup>
count aloud	Count by tens from 10 to 100. Count by hundreds from 100 to 1000.
mental	a. Addition: 3 + 3
math	<b>b. Addition:</b> 30 + 30
	<b>c. Addition:</b> 300 + 300
	<b>d. Addition:</b> 40 + 50
	e. Addition: 200 + 600
	f. Money: 50¢ + 50¢
	<b>g. Money:</b> 20¢ + 20¢ + 20¢
	<b>h. Addition:</b> 500 + 500 + 500
problem	Fill in the missing numbers:
solving	17, 15, 13,,, 5, 3, 1
	Focus Strategy: Find a Pattern
	<b>Understand</b> We are given a list of numbers. Some of the

numbers are missing. We are asked to find the missing numbers.

**Plan** We will *find a pattern.* We see that the numbers "count down," or decrease, from left to right. We look for a "counting down" pattern to help us find the missing numbers.

**Solve** We notice that the numbers decrease by twos. The second number, 15, is two less than the first number. The third number, 13, is two less than 15.

<sup>&</sup>lt;sup>1</sup> For instructions on how to use the Power Up activities, please consult the preface.

On the right, we see that the number 3 is two less than 5, and that the number 1 is two less than 3.

The pattern is "count down by twos." Two less than 13 is 11, two less than 11 is 9, and two less than 9 is 7. So the missing numbers are **11, 9,** and **7.** 

**Check** We know our answer is reasonable because each number we found is two less than the previous number in the list, which fits the pattern we found.

New Concepts

Sequences

Counting is a math skill that we learn early in life. Counting by ones, we say the numbers

1, 2, 3, 4, 5, 6, ...

#### **Reading Math**

The three dots mean that the sequence continues even though the numbers are not written. These numbers are called **counting numbers**. We can also count by a number other than one. Below we show the first five numbers for counting by twos and the first five numbers for counting by fives.

> 2, 4, 6, 8, 10, ... 5, 10, 15, 20, 25, ...

An ordered list of numbers forms a **sequence**. Each member of the sequence is a **term**. We can study a sequence to discover its counting pattern, or rule. The rule can be used to find more terms in the sequence.

**Connect** What is another way to describe the rule of each sequence?

#### What are the next three terms in this counting sequence?

3, 6, 9, 12, \_\_\_\_, \_\_\_, ...,

The pattern is "count up by threes." To find the next three terms, we may count up by threes, or we may count up by ones and emphasize every third term (one, two, *three,* four, five, *six,* ...). Either way, we find that the next three terms are **15, 18,** and **21.** 

**Example 2** 

Example 1

Describe the rule for the counting sequence below. What is the next term in the sequence?

56, 49, 42, \_\_\_\_, ...

	This sequence counts is <b>"count down by se</b> gives us <b>35.</b> <b>Represent</b> Write a se sixes."	evens." Counting do	
Digits	5, 6, 7, 8, and 9. The digit is 5. The numb the last digit is 4.	e number 385 has th	em. They are 0, 1, 2, 3, 4, hree digits, and the last has twelve digits, and
Example	The number 186,000		
	The number 186,000 h	has <b>six digits.</b>	
Example		_	······································
LAdimpie	What is the last digit		
	The number 26,348 ha	as five digits. The la	st digit is <b>8.</b>
Lesson Practice		be the rule for each terms in the sequer	counting sequence. Then nce.
	<b>a.</b> 6, 8, 10,, _	,,	
	<b>b.</b> 7, 14, 21,,	,,	
	<b>c.</b> 4, 8, 12,, _	,,	
	<b>d.</b> 21, 18, 15,		
	<b>e.</b> 45, 40, 35,		
	<b>f.</b> 12, 18, 24,		
		e in each of these n	
	<b>g.</b> 36,756	<b>h.</b> 8002	i. 1,287,495
	What is the last digi		
	j. 17	<b>k.</b> 3586	I. 654,321
	down by nines.	rite a sequence usin ."	g the rule count

Distributed and Integrated Written Practice **Connect**) Write the next term in each counting sequence: **\*1.** 10, 15, 20, \_\_\_\_, ... **\*2.** 56, 49, 42, \_\_\_\_, ... **\*3.** 8, 16, 24, \_\_\_\_, ... **\*4.** 18, 27, 36, 45, \_\_\_\_, ... **\*5.** 24, 21, 18, \_\_\_\_, ... **\*6.** 32, 28, 24, 20, \_\_\_\_, ... **Connect**) Write the missing term in each counting sequence: **\*7.** 7. 14. \_\_\_\_, 28. 35. ... **\*8.** 40. \_\_\_\_, 30. 25. 20. ... **\*9.** 20, \_\_\_\_, 28, 32, 36, ... **\* 10.** 24, 32, \_\_\_\_, 48, ... **\*12.** 21, 28, \_\_\_\_, 42, ... **\* 11.** \_\_\_\_, 36, 30, 24, ... Generalize) Describe the rule for each counting sequence, and write the next three terms. **\* 13.** 3, 6, 9, 12, \_\_\_\_, \_\_\_, \_\_\_, .... **\*14.** 8, 16, 24, \_\_\_\_, \_\_\_\_, ...., ...., ...., ...., **\* 15.** 6, 12, 18, \_\_\_\_, \_\_\_\_, \_\_\_\_, .... **\*16.** 40, 35, 30, \_\_\_\_, \_\_\_\_, ...., **\* 17.** 18, 21, 24, \_\_\_\_, \_\_\_\_, ...., \_\_\_\_, .... **\*18.** 9, 18, 27, \_\_\_\_, \_\_\_\_, \_\_\_\_, .... **19.** What word names an ordered list of numbers? How many digits are in each number? **20.** 186,000 **21.** 73.842 22. 30.004.091 **Classify** What is the last digit of each number? \*23. 26,348 \*24. 347 **\*25.** 9,675,420

<sup>\*</sup> Beginning in this lesson, we star the exercises that cover challenging or recently presented content. We encourage students to work first on the starred exercises with which they might want help, saving the easier exercises for last.



### • Even and Odd Numbers

Power Up

facts	Power Up A						
count aloud	Count up and down by tens between 10 and 100. Count up and down by hundreds between 100 and 1000.						
mental	a. Addition: 6 + 6						
math	<b>b. Addition:</b> 60 + 60						
	<b>c. Addition:</b> 600 + 600						
	d. Time: 60 seconds + 70 seconds						
	e. Time: 70 seconds + 80 seconds						
	f. Addition: 300 + 300 + 300						
	<b>g. Addition:</b> 90 + 90						
	<b>h. Money:</b> 50¢ + 50¢ + 50¢						
problem solving	Choose an appropriate problem-solving strategy to solve this problem. Draw the missing shapes in this sequence. Then describe the sequence in words.						
	,,,,,,,						
New Concept							
	Whole numbers are the counting numbers and the number 0.						
	0, 1, 2, 3, 4, 5, 6,						
	Counting by twos, we say the numbers						
	2, 4, 6, 8, 10, 12, 14, 16, 18, 20,						

#### **Thinking Skill**

Connect

Why do even numbers continue without end? This is a special sequence. The numbers on the previous page are **even numbers.** The number 0 is also an even number. The sequence of even numbers continues without end. The numbers 36 and 756 and 148,567,896,094 are all even. We can tell whether a whole number is even by looking at the last digit of the number. If the last digit is even, then the number is even. So even numbers end with 0, 2, 4, 6, or 8.

An even number of objects can be arranged in pairs. Twelve is an even number. Here we show 12 dots arranged in six pairs. Notice that every dot has a partner.

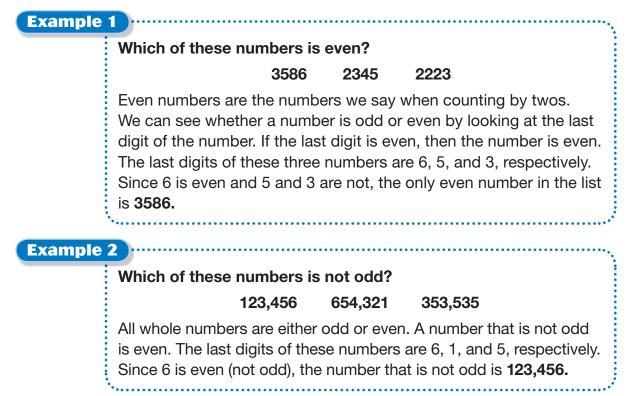
Next we show 13 dots arranged in pairs. We find that there is a dot that does not have a partner. So 13 is not even.



The whole numbers that are not even are **odd**. We can make a list of odd numbers by counting up by twos from the number 1. Odd numbers form this sequence:

1, 3, 5, 7, 9, 11, 13, 15, 17, ...

If the last digit of a number is 1, 3, 5, 7, or 9, then the number is odd. All whole numbers are either odd or even.



Half of an even number is a whole number. We know this because an even number of objects can be separated into two equal groups. However, half of an odd number is not a whole number. If an odd number of objects is divided into two equal groups, then one of the objects will be split in half.

These two word problems illustrate dividing an even number in half and dividing an odd number in half:

Sherry has 6 apples to share with Leticia. If Sherry shares the apples equally, each girl will have 3 apples.

Herman has 5 apples to share with Ivan. If Herman shares the apples equally, each boy will have  $2\frac{1}{2}$  apples.



#### Halves

The table below lists the counting numbers 1 through 10. Below each counting number we have recorded half of the number. Continue the list of counting numbers and their halves for the numbers 11 through 20.

Counting Number	1	2	3	4	5	6	7	8	9	10
Half of Number	<u>1</u> 2	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	5

**Discuss** Is the top number double the bottom number? Explain?

Lesson Practice

**Classify** Describe each number as odd or even:

<b>a.</b> 0	<b>b.</b> 1234	<b>c.</b> 20,001
<b>d.</b> 999	<b>e.</b> 3000	<b>f.</b> 391,048

- **Explain** All the students in the class separated into two groups. The same number of students were in each group. Was the number of students in the class an odd number or an even number? Explain why.
- h. Tamayo has seven berries to share with Kasim. If Tamayo shares the berries equally, how many berries will each person have?

#### Thinking Skill

#### Discuss

If Herman were sharing trading cards, would the answer still be  $2\frac{1}{2}$ ? Why or why not?

Written Pra	Dettee D	istributed an	d Integrated			
* <sup>1</sup> 1. Generalize	If a whole num	nber is not	even, then wh	nat is it?		
What is the last d	git of each nu	mber?				
<b>2.</b> 47,286,560		<b>3.</b> 296,317				
Classify Describ	e each numbe	er as odd o	or even:			
* <b>4.</b> 15	*	<b>*5.</b> 196		* <b>6.</b> 3567		
7. Which of the	se numbers is	even?				
	3716	2345	2223			
<b>8.</b> Which of the	se numbers is	odd?				
	45,678	56,789	67,890			
9. Which of the	se numbers is	not odd?				
	333,456	654,321	353,535			
<b>10.</b> Which of thes	se numbers is	not even?				
	300	232	323			
<b>Conclude</b> Write	the next three	terms in e	each counting	sequence:		
* <b>11.</b> 9, 12, 15,	,,,					
<b>* 12.</b> 16, 24, 32,	,,	.,				
<b>* 13.</b> 120, 110, 100	),,, _	,				
* <b>14.</b> 28, 24, 20,	,,	., · · ·				
<sup>1</sup> The itali	cized numbers wit	hin narenthes	es underneath ea	ch problem number are called less		

<sup>&</sup>lt;sup>1</sup> The italicized numbers within parentheses underneath each problem number are called *lesson reference numbers*. These numbers refer to the lesson(s) in which the major concept of that particular problem is introduced. If additional assistance is needed, refer to the discussion, examples, or practice problems of that lesson.

<b>* 15.</b> 55, 50, 45,,	,,	<b>* 16.</b> 18, 27, 3	36,,,,
<b>* 17.</b> 36, 33, 30,,	,,	<b>* 18.</b> 18, 24, 3	30,,,
<b>* 19.</b> 14, 21, 28,,	,,	<b>* 20.</b> 66, 60, 5	54,,,
<b>*21.</b> 48, 44, 40,,	,,	* <b>22.</b> 99, 90, 8	81,,,
<b>*23.</b> 88, 80, 72,,	,,	* <b>24.</b> 84, 77, 7	70,,,
	e All the students ir students were in ea total number of students	ch line. Which of	the following
<b>A</b> 30	<b>B</b> 31	<b>C</b> 32	<b>D</b> 28
<b>26.</b> What number is $I_{(2)}$	half of 5?		
*27. Multiple Choic		umbers is a whole	e number? Draw a
A half of 11	B half of 12	C half of 13	<b>D</b> half of 15

Use this table to answer problems 28-30:

Number of Tickets	1	2	3	4
Cost	\$7	\$14	\$21	\$28

**28.** Describe the relationship between the number of tickets and the cost.

\*29. Generalize Write a rule that describes how to find the cost of any number of tickets.

**30. Predict** What is the cost of 10 tickets?

# • Using Money to Illustrate Place Value

Power Up	
facts	Power Up A
count aloud	Count up and down by tens between 10 and 200. Count up and down by hundreds between 100 and 2000.
mental	<b>a. Money:</b> 30¢ + 70¢
math	<b>b. Addition:</b> 20 + 300
	<b>c. Addition:</b> 320 + 20
	<b>d. Addition:</b> 340 + 200
	<b>e. Addition:</b> 250 + 40
	f. Addition: 250 + 400
	g. Time: 120 seconds + 60 seconds
	h. Addition: 600 + 120
problem solving	How many two-digit counting numbers are there?
	Focus Strategy: Use Logical Reasoning
	<b>Understand</b> The counting numbers are the numbers we say when we count up by 1s (1, 2, 3, 4, and so on). We are asked to find the number of two-digit counting numbers.
	<b>Plan</b> We could list all the counting numbers with two digits and then count the numbers in our list, but that would take too long. Instead, we will <i>use logical reasoning</i> to solve the problem. We will use information we know to find the information we are asked for in the problem.

**Solve** We know that the greatest two-digit counting number is 99. The next counting number, 100, contains three digits. Suppose we listed all the counting numbers from 1 to 99. That would be just like counting from 1 to 99, so we know there are 99 counting numbers from 1 to 99.

Remember that we are asked to find the number of **two-digit** counting numbers. How many of the numbers from 1 to 99 contain exactly two digits? We know there are 9 counting numbers that have only one digit (the numbers 1, 2, 3, ..., 7, 8, 9). So there are 99 - 9, or 90 counting numbers that contain exactly two digits.

**Check** We found that there are 90 two-digit counting numbers. We know our answer is reasonable because there are 99 counting numbers from 1 to 99, and nine of those numbers (the numbers 1–9) contain only one digit. By using logical reasoning, we found the answer more quickly than if we had listed and counted all the two-digit counting numbers.





Visit www. SaxonMath.com/ Int5Activities for a calculator activity. Each digit in a number has a **place value**. The value of a digit depends on its place, or position, in the number. We identify the value of the digits in a number when we write the number in **expanded form**. Expanded form is a way of writing a number that shows the value of each digit. We can use money to illustrate place value.

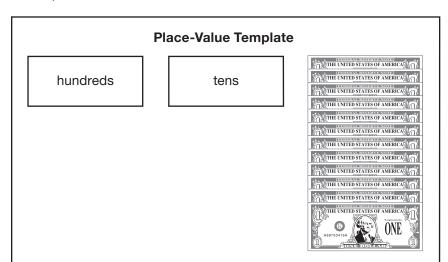


# Place Value

Materials needed:

- money manipulatives from Lesson Activities 1, 2, and 3
- Lesson Activity 8
- locking plastic bag
- 3 paper clips

**Model** Place twelve \$1 bills on the template in the ones position, as shown below.



We can use fewer bills to represent \$12 by exchanging ten \$1 bills for one \$10 bill. Remove ten \$1 bills from the template, and replace them with one \$10 bill in the tens position. You will get this arrangement of bills:

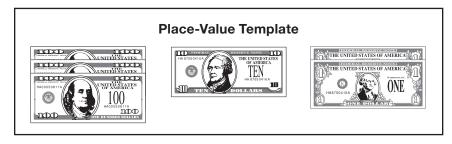


The bills on the template illustrate the expanded form of the number 12.

Expanded form: 1 ten + 2 ones

10 + 2

Now place \$312 on the place-value template, using the fewest bills necessary. Use the bills to write 312 in expanded form.



From the template we see the expanded form of 312.

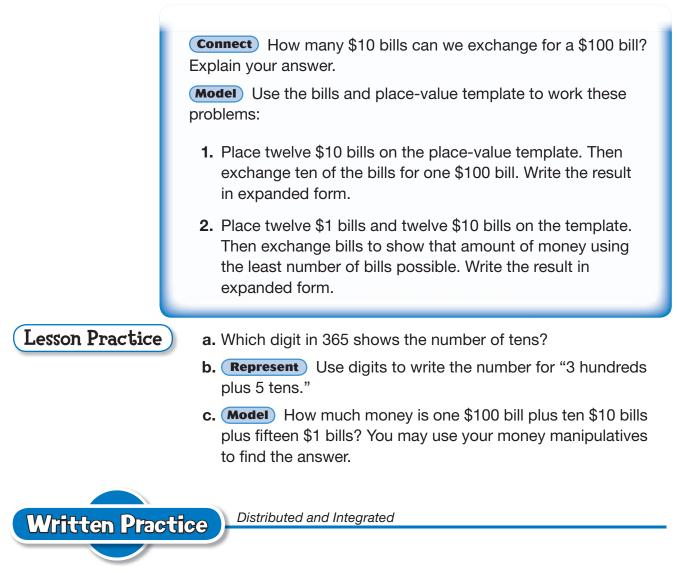
3 hundreds + 1 ten + 2 ones

300 + 10 + 2

## Thinking Skill

Discuss

Why can we exchange 10 ones for 1 ten?



- **1. Represent** Use digits to write the number for "5 hundreds plus <sup>(3)</sup> 7 tens plus 8 ones."
- **2. Represent** Use digits to write the number for "2 hundreds plus <sup>(3)</sup> 5 tens plus 0 ones."
- **3.** In 560, which digit shows the number of tens?
- **4.** In 365, which digit shows the number of ones?
- **5.** Ten \$10 bills have the same value as one of what kind of bill?
- **6.** The greatest two-digit odd number is 99. What is the greatest two-digit even number?

*7. Multiple Choice	Which of these	e numbers is <i>not</i>	even?
<sup>(2)</sup> <b>A</b> 1234	<b>B</b> 2345	<b>C</b> 3456	<b>D</b> 4560
*8. Multiple Choice	Which of these	e numbers is <i>not</i>	odd?
<b>A</b> 365	<b>B</b> 653	<b>C</b> 536	<b>D</b> 477
*9. Multiple Choice		•	
<b>A</b> 22	<b>B</b> 25	<b>C</b> 50	<b>D</b> 38
*10. Multiple Choice		to 12 by 2s or by	/ 3s. We do not count
<b>A</b> 1s	<b>B</b> 4s	<b>C</b> 5s	D 6s
<b>Conclude</b> Write the n	ext three terms	in each counting	y sequence:
<b>11.</b> 9, 12, 15,,	,,		
<b>12.</b> 54, 48, 42,,	,,		
<b>13.</b> 8, 16, 24,,	,,		
<b>14.</b> 80, 72, 64,,	,,		
<b>15.</b> 16, 20, 24,,	,,		
<b>16.</b> 40, 36, 32,,	,, · · ·		
<b>Generalize</b> Describe three terms.	the rule for each	n counting seque	ence, and find the next
<b>17.</b> 27, 36, 45,,	,,		
<b>18.</b> 81, 72, 63,,	,,		
<b>19.</b> 10, 20, 30,,	,,		
<b>20.</b> 33, 30, 27,,	,,		

**21.** What number equals four tens? (3)

- **22.** What number equals five hundreds?
- **23.** Model How much money is two \$100 bills plus twelve \$10 bills plus fourteen \$1 bills? You may use your money manipulatives to find the answer.
- **24.** The number 80 means "eight tens." The number 800 means eight what?
- \*25. **Predict** The fifth term in the counting sequence below is 20. What is the ninth term in this sequence?

4, 8, 12, 16, ...

- **26.** How much money is half of \$10?
- 27. How much money is half of \$5?
- **\*28.** (2) Is the greatest two-digit number an odd number or an even number? How do you know?

Use this table to answer problems 29 and 30:

Number of Tricycles	1	2	3	4
Number of Wheels	3	6	9	12

- **29. Generalize** Write a rule that describes how to find the number of tricycles for any number of wheels.
- **30.** How many tricycles are represented by 27 wheels?



# Comparing Whole Numbers

PowerUp

facts	Power Up A
count aloud	Count up and down by tens between 0 and 200. Count up and down by hundreds between 0 and 2000.
mental math	<ul> <li>a. Money: 300¢ + 300¢ + 20¢ + 20¢</li> <li>b. Money: 250¢ + 50¢</li> <li>c. Addition: 300 + 350</li> <li>d. Addition: 320 + 320</li> <li>e. Addition: 300 + 300 + 50 + 50</li> <li>f. Money: 250¢ + 60¢</li> <li>g. Addition: 340 + 600</li> <li>h. Addition: 240 + 320</li> </ul>
problem solving	The two-digit counting numbers that contain the digits 1 and 2 are 12 and 21. There are six three-digit counting numbers that contain the digits 1, 2, and 3. One of these numbers is 213. What are the other five numbers? Focus Strategy: Make an Organized List

**Understand** We look for the information that is given. We are told that there are six three-digit counting numbers that contain the digits 1, 2, and 3. One of those numbers is 213. We are asked to find the other five three-digit counting numbers that contain the digits 1, 2, and 3.

**Plan** We want to use a problem-solving strategy that helps us quickly find the answer in a way that is understandable and organized. We will *make an organized list* to do this.

We can organize our list starting with the first digit of the counting numbers we are looking for. First we will list all the possibilities that begin with the digit 1, then all the possibilities that begin with the digit 2, and then all the possibilities that begin with the digit 3.

**Solve** If the first digit is 1, then there are two possible counting numbers that satisfy the conditions of the problem: 123 and 132. If the first digit is 2, the possibilities are 213 and 231. If the first digit is 3, the possibilities are 312 and 321. Our list might look like this:

123	213	312
132	231	321

The number 213 was given to us in the problem. We are asked for the other five three-digit counting numbers that contain the digits 1, 2, and 3. They are **123**, **132**, **231**, **312**, and **321**.

**Check** We know that our answer is reasonable because each number contains the digits 1, 2, and 3. Making an organized list helped us make sure that we found all the numbers.



When we count from 1 to 10, we count in order from least to greatest.

```
1, 2, 3, 4, 5, 6, 7, 8, 9, 10

least greatest
```

Of these numbers, the least is 1 and the greatest is 10. Since these numbers are arranged in order, we can easily see that 5 is greater than 4 and that 5 is less than 6.

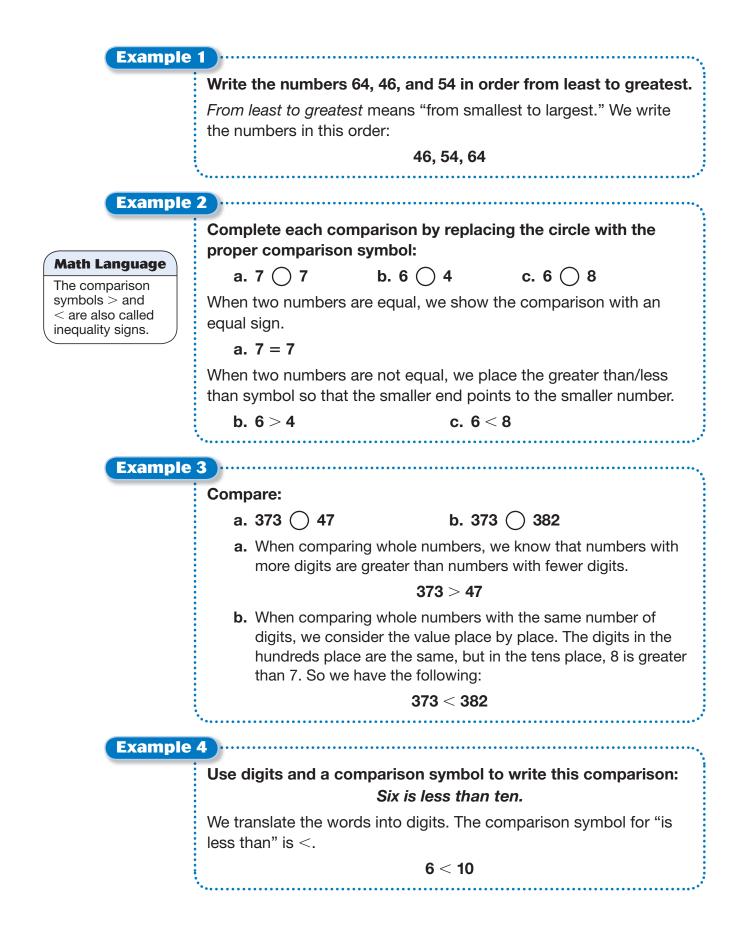
We can use mathematical symbols to compare numbers. **Comparison symbols** include the equal sign (=) and the greater than/less than symbol (> or <).

5 = 5 is read	"Five is <b>equal to</b> five."
5 > 4 is read	"Five is greater than four."
5 < 6 is read	"Five is less than six."

When using a greater than/less than symbol to compare two numbers, we place the symbol so that the smaller end points to the smaller number.

#### Math Language

An equal sign is used to show that two quantities are equal.



Lesson Practice

a. Write the numbers 324, 243, and 423 in order from least to greatest.

Complete each comparison by replacing the circle with the correct comparison symbol:

**b.** 36 () 632

) 90

**d.** 90 (

**c.** 110 () 101 **e.** 112 () 121

**Represent** Write each comparison using digits and a comparison symbol:

- f. Twenty is less than thirty.
- g. Twelve is greater than eight.

Written Practice

Distributed and Integrated

**Represent** Write each comparison using digits and a comparison symbol:

- **1.** Four is less than ten.
- **2.** Fifteen is greater than twelve.

Complete each comparison by replacing the circle with the correct comparison symbol:

- **3.** 97 () 101 **4.** 34 () 43
- **5. Represent** Use digits to write the number for "3 hundreds plus 6 tens plus 5 ones."
- 6. Which digit in 675 shows the number of hundreds?
- 7. Which digit in 983 shows the number of ones?
- 8. One \$100 bill equals ten of what kind of bill?

**Classify** Describe each number as odd or even:

- \***9.** 36,275 \***10.** 36,300 \***11.** 5,396,428
- **12. Connect** The greatest two-digit odd number is 99. What is the greatest three-digit odd number?

**13. Multiple Choice** We can count to 18 by 2s or by 3s. We do not count <sup>(1)</sup> to 18 when counting by

**A** 1s **B** 4s **C** 6s **D** 9s

- **14.** Write the numbers 435, 354, and 543 in order from least to greatest.
- **15. Predict** The fourth term in the counting sequence below is 24. What <sup>(1)</sup> is the ninth term in this sequence?

6, 12, 18, ...

\*16. Model What is the value of five \$100 bills, thirteen \$10 bills, and (3) ten \$1 bills? You may use your money manipulatives to find the answer.

**Conclude** Write the next three terms in each counting sequence:

 17. 20, 24, 28, \_\_\_\_, \_\_\_\_, \_\_\_\_, ...

 18. 106, 104, 102, \_\_\_\_, \_\_\_\_, ...,

 19. 0, 6, 12, \_\_\_\_\_, \_\_\_\_, ...,

 20. 0, 7, 14, \_\_\_\_\_, \_\_\_\_, ...,

 21. 40, 32, 24, \_\_\_\_\_, \_\_\_\_, ...,

 22. 45, 36, 27, \_\_\_\_\_, \_\_\_\_\_, ...,

 23. What number equals 9 tens?

 24. What number equals 11 tens?

 (3)

 Predict
 What is the seventh term in this counting sequence?

 (1)

 (2)

 Predict

 Is the eleventh term of this counting sequence odd or

veven?

2, 4, 6, 8, ...

# **27.** What number is half of 9?

**\*28.** (2) In Room 12 there is one more boy than there are girls. Is the number of students in Room 12 odd or even? How do you know?

Use this table to answer problems 29 and 30:

Number of Ladybugs	1	2	3	4
Number of Legs	6	12	18	24

**29.** (1) Generalize Write a rule that describes how to find the number of ladybugs for any number of legs.

30. How many ladybugs are represented by 54 legs?



The chart below shows a list of animals and the number of teeth each animal has.

Real-World Connection

- **a.** Order the numbers from least to greatest.
- **b.** Write a comparison of the number of teeth cats and ferrets have using digits and a comparison symbol.
- c. Then write the same comparison using words.

Animal	Number of Teeth		
Alligator	76		
Cat	30		
Dog	42		
Elephant	26		
Ferret	34		
Horse	40		



# Naming Whole Numbers and Money

Power Up A
Count up and down by tens between 0 and 200. Count up and down by hundreds between 0 and 2000.
<b>a. Addition:</b> 200 + 60 + 300
<b>b. Addition:</b> 20 + 600 + 30
<b>c. Money:</b> 350¢ + 420¢
<b>d. Measurement:</b> 250 cm + 250 cm
<b>e. Addition:</b> 400 + 320 + 40
f. Addition: 30 + 330 + 100
g. Addition: 640 + 250
h. Addition: 260 + 260
Choose an appropriate problem-solving strategy to solve this problem. Write all the three-digit numbers that each have the digits 2, 3, and 4. Arrange the numbers in order from least to greatest.



In this lesson we can use place value to help name numbers. In order to name larger numbers, we should first be able to name numbers that have three digits. Let's consider the number 365. Below we use expanded form to break the number into its parts. Then we show the name of each part.

```
three hundreds + six tens + five ones
"three hundred" "sixty" "five"
```

We will use words to name a number that we see and use digits to write a number that is named. Look at these examples:

- 18 eighteen
- 80 eighty
- 81 eighty-one
- 108 one hundred eight
- 821 eight hundred twenty-one

Notice that we do not use the word *and* when naming whole numbers. For example, we write the number 108 as "one hundred eight," not "one hundred *and* eight." Also notice that we use a hyphen when writing the numbers from 21 to 99 that do not end in zero. For example, we write 21 as "twenty-one," not "twenty one."

Example 1

The land area of Cameron County, Texas, is nine hundred six square miles. The land area of Collingsworth County, Texas, is nine hundred nineteen square miles. Which county has the greater land area?

Since 919 square miles is greater than 906 square miles,

**Collingsworth County** has the greater land area.

Dollars and cents are written with a dollar sign and a **decimal point.** To name an amount of money, we first name the number of dollars, say "and," and then name the number of cents. The decimal point separates the number of dollars from the number of cents. For example, \$324.56 is written as "three hundred twenty-four dollars and fifty-six cents."

#### Example 2

The cost of fuel to heat a home for five months is shown below. Order the months from most expensive to least expensive.

Month	Cost
November	\$141
December	\$315
January	\$373
February	\$264
March	\$149

By comparing the dollar amounts, we can arrange these five months in order from most expensive to least expensive.

### January, December, February, March, November

Lesson Practice

- a. Use words to name \$563.45.
- **b.** Use words to name 101.
- c. Use words to name 111.
- d. Use digits to write two hundred forty-five.
- e. Use digits to write four hundred twenty.
- f. Use digits to write five hundred three dollars and fifty cents.
- **g.** In 1825 the Erie Canal consisted of eighty-three locks. A reconstruction completed in 1862 changed the number of locks to seventy-two. During which year, 1825 or 1862, did the Erie Canal contain the greater number of locks?
- **h.** This table shows the total sales at a school bookstore during one week:

Day	Total Sales
Monday	\$40
Tuesday	\$26
Wednesday	\$18
Thursday	\$25
Friday	\$11

Order the total sales amounts from least to greatest.

Written Practice

Distributed and Integrated

- \*1. **Represent** Use digits to write three hundred seventy-four dollars and twenty cents.
- \*2. **Represent** Use words to name \$623.15.
  - **3. Represent** Use digits to write two hundred five.
  - **4.** Use words to name 109.

**5. Represent** Write this comparison using digits and a comparison symbol: One hundred fifty is greater than one hundred fifteen.

6. Compare: 346 () 436

**7. Represent** Use digits to write the number for "5 hundreds plus 7 tens <sup>(3)</sup> plus 9 ones."

**\*8. Analyze** Arrange these four numbers in order from least to greatest:

462 624 246 426

- 9. Which digit in 567 shows the number of tens?
- 10. When counting up by tens, what number comes after 90?
- **Classify** Describe each number as odd or even:
- \* **11.** 363,636 **12.** 36,363 **13.** 2000
- \* **14.** The greatest three-digit odd number is 999. What is the greatest three-digit even number?
  - **15. Multiple Choice** We can count to 20 by 2s or by 10s. We do not count to 20 when counting by
    - **A** 1s **B** 3s **C** 4s **D** 5s
  - **16. Multiple Choice** There are equal numbers of boys and girls in the room. Which of the following could *not* be the number of students in the room?

**A** 12 **B** 29 **C** 30 **D** 44

\* **17. Model** What is the value of six \$100 bills, nine \$10 bills, and twelve \$1 bills? You may use your money manipulatives to help find the answer.

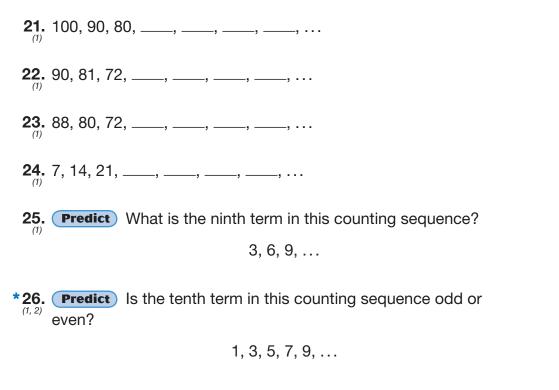
**Conclude** Write the next four terms in each counting sequence:

**18.** 0, 9, 18, \_\_\_\_, \_\_\_, \_\_\_, \_\_\_, ...

**19.** 25, 30, 35, \_\_\_\_, \_\_\_\_, \_\_\_\_, \_\_\_\_, ...

**20.** 6, 12, 18, \_\_\_\_, \_\_\_, \_\_\_, \_\_\_, ...,

**Generalize** State the rule for each counting sequence, and find the next four terms.



27. Is the greatest three-digit whole number odd or even?

**28.** (2) **Explain** Sean and Jerry evenly shared the cost of a \$7 pizza. How much did each person pay? Explain how you know.

Use this table to answer problems 29 and 30:

Number of Dollars	1	2	3	4
Number of Quarters	4	8	12	16

- **29. Generalize** Write a rule that describes how to find the number of quarters for any number of dollars.
- **30.** What number of quarters represents \$10?

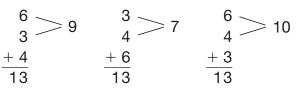
# Adding Whole Numbers

Power Up

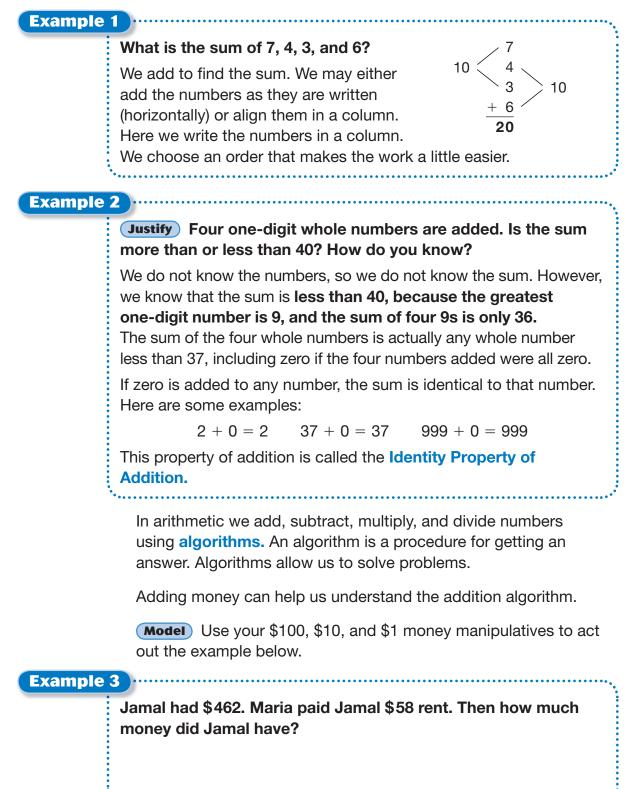
facts	Power Up A
count aloud	Count up and down by 20s between 0 and 200. Count up and down by 200s between 0 and 2000.
mental math	<ul> <li>a. Addition: 400 + 50 + 300 + 40</li> <li>b. Addition: 320 + 300</li> <li>c. Addition: 320 + 30</li> <li>d. Addition: 320 + 330</li> <li>e. Addition: 60 + 200 + 20 + 400</li> <li>f. Addition: 400 + 540</li> <li>g. Money: \$40 + \$250</li> <li>h. Measurement: 450 yards + 450 yards</li> </ul>
problem solving	Choose an appropriate problem-solving strategy to solve this problem. Dave purchased milk from the vending machine for 60¢. He used 6 coins. As Dave inserted the coins into the machine, the display counted up as follows: 5¢, 30¢, 35¢, 45¢, 55¢, 60¢. What coins did Dave use to purchase the milk?



Numbers that are added are called **addends**. The answer to an addition problem is the **sum**. We may add numbers in any order to find their sum. For example, 5 + 6 gives us the same sum as 6 + 5. This property of addition is called the **Commutative Property of Addition**. When adding more than two numbers, this property allows us to add in any order we choose. On the next page we show three ways to add 6, 3, and 4. We point out the two numbers we added first.



As shown in the last example, we can sometimes find pairs of numbers that add up to 10. This makes the addition easier.



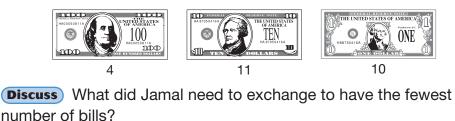
First we will use bills to model the problem: Jamal had \$462.



Maria paid Jamal \$58 rent.



When Jamal added Maria's rent money to the money he already had, he ended up with four \$100 bills, eleven \$10 bills, and ten \$1 bills.



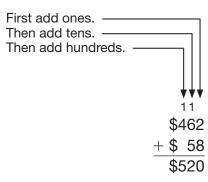
**Conclude** How much money did Jamal have after Maria paid him for rent?

Now we will show a pencil-and-paper solution that uses the addition algorithm. When using this addition algorithm, we are careful to line up digits that have the same place value.

Jamal had \$462.	\$462		
Maria paid Jamal \$58.	+ \$ 58		

Then Jamal had ...

First we add the ones, then the tens, and then the hundreds.



Notice we exchange 10 ones for 1 ten. Then we exchange 10 tens for 1 hundred.

**Connect** How are these exchanges similar to paper-money exchanges?



Find each sum. When adding, look for combinations of numbers that add up to 10.

<b>a.</b> 8 + 6 + 2	<b>b.</b> 4 + 7 + 3 + 6
<b>c.</b> 9 + 6 + 4	<b>d.</b> 4 + 5 + 6 + 7
<b>e.</b> 7 + 3 + 4	<b>f.</b> 2 + 6 + 3 + 5
<b>g.</b> 6 + 7 + 5	<b>h.</b> 8 + 7 + 5 + 3

i. **Multiple Choice** The sum of 5 one-digit whole numbers is certain to be \_\_\_\_\_.

A greater than 4	<b>B</b> less than 50
<b>C</b> an odd number	<b>D</b> an even number

Use the addition algorithm to find each sum. When putting the numbers into columns, remember to line up the last digits.

j. \$463 + \$158	<b>k.</b> 674 + 555	I. \$323 \$142 + \$365	
<b>m.</b> 543 + 98	<b>n.</b> \$47 + \$485	- 4000	

Distributed and Integrated

\*1. Model You may use money manipulatives to answer the question in this word problem:

Iggy had \$520. After Hannah paid him \$86 rent, how much money did Iggy have?

\*2. **Represent** Use words to name \$212.50.

3. In the number 274, which digit shows the number of hundreds?

**Classify** Describe each number as odd or even:

**4.** 1234 **5.** 12,345 **6.** 1,234,567

7. Use digits to write five hundred eight dollars.

8. Use words to name 580.

Written Practice

Find each sum. Look for combinations of 10.

9. $1 + 6 + 9$		<b>10.</b> $7 + 6 + 4$	
<b>11.</b> $8 + 3 + 1 + 7$		<b>12.</b> $4 + 5 + 6 + 7$	
<b>13.</b> \$436 + \$527	<b>14.</b> 592 (6) + 408	<b>15.</b> 963 (6) <u>+ 79</u>	<b>16.</b> \$180 + \$747

**17. Multiple Choice** All the books were put into two piles. There was one more book in one pile than in the other pile. The total number of books in both piles could *not* be \_\_\_\_\_.
 **A** 28 **B** 29 **C** 33 **D** 55

**Predict** Find the eighth term in each counting sequence:

- **18.** 10, 20, 30, ...**19.** 6, 12, 18, ...**20.** 7, 14, 21, ...**21.** 8, 16, 24, ...
- **22.** Compare: nine hundred sixteen  $\bigcirc$  nine hundred sixty

**23. Represent** Write this comparison using digits and a comparison symbol:

Six hundred ninety is greater than six hundred nine.

- **\*24.** Analyze Compare:  $5 + 5 + 5 \bigcirc 4 + 5 + 6$ 
  - **25.** The smallest even two-digit whole number is 10. What is the smallest <sup>(2)</sup> odd two-digit whole number?
- \*26. Analyze Is the smallest three-digit number odd or even?
  - **27.** Predict Is the 29th term in this counting sequence odd or even? Explain how you know.

2, 4, 6, 8, ...

\*28. Analyze Tabitha needs to read nine pages in her history book. If she wants to read half of those pages before dinner, how many pages does she need to read?

Use this table to answer problems 29 and 30:

Number of Quarters	1	2	3	4
Number of Nickels	5	10	15	20

- **29. Generalize** Write a rule that describes how to find the number of quarters for any number of nickels.
- \*30. How many quarters are represented by fifty nickels?

Einishere Real-World Connection Darius had \$356 in his savings account. He earned \$64 and deposited it in his account. How much money is in his account now?

- **a.** Use money manipulatives to model the problem. Which bills did Darius need to exchange to have the fewest number of bills?
- **b.** Show how to solve the problem using the addition algorithm.



- Writing and Comparing Numbers Through Hundred Thousands
- Ordinal Numbers



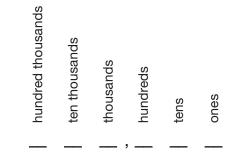
count aloud	Count up and down by 20s between 0 and 200. Count up and down by 200s between 0 and 2000.	
mental	<b>a. Money:</b> \$25 + \$25	
math	<b>b. Money:</b> \$300 + \$450	
	<b>c. Money:</b> \$250 + \$250	
	<b>d. Addition:</b> 30 + 450	
	<b>e. Money:</b> \$75 + \$25	
	f. Money: \$750 + \$250	
	<b>g. Money:</b> \$50 + \$350	
	h. Time: 360 seconds + 360 seconds	
problem solving	Choose an appropriate problem-solving strategy to solve this problem. The sum of 12 and 21 is 33. What is the sum of the sit three-digit numbers that each have the digits 1, 2, and 3? If the six numbers are arranged vertically, what is the sum of the digit in each column? Why is the sum of the digits in each column the same?	
New Concepts		

We have practiced naming whole numbers with three or fewer digits. In this lesson we will begin naming whole numbers with four, five, and six digits.

# Writing and Comparing Numbers Through Hundred Thousands

#### **Reading Math**

Our place-value system is a **base-ten system.** Each place value is 10 times greater than the place value to its right. The value of a digit depends upon its position in a number. The following chart lists the values of the first six whole-number places.



**Discuss** Describe the relationship between the thousands place and the hundreds place.

Commas are often used to write a whole number with many digits so that the number is easier to read. To place commas in a whole number, we count digits from the right-hand end of the number and insert a comma after every three digits.

54,321

The comma in this number marks the end of the thousands. To name this number, we read the number formed by the digits to the left of the comma and then say "thousand" at the comma. Finally, we read the number formed by the last three digits.

54,321

fifty-four thousand, three hundred twenty-one

Notice that we place a comma after the word *thousand* when we use words to name a number. Here we show some other examples:

- \$27,050 twenty-seven thousand, fifty dollars
- 125,000 one hundred twenty-five thousand
- 203,400 two hundred three thousand, four hundred

Whole numbers with four digits may be written with a comma, but in this book, four-digit whole numbers will usually be written without a comma.

#### Example 1

# Use words to name 52370.

To help us read the number, we write it with a comma:

52,370

We name the number formed by the digits in front of the comma, write "thousand" and a comma, and then name the number formed

i

	by the digits after the comma. So 52,370 is <b>fifty-two thousand,</b>						
	three hundred seventy.						
-							

**Justify** Why didn't we place the comma between the 3 and the 7? Explain your answer.

Exam	ple 2	•

Use digits to write "one hundred fifty thousand, two hundred thirty-four."

We use digits to write "one hundred fifty" and write a comma for the word *thousand*. Then we use digits to write "two hundred thirty-four."

### 150,234

**Example 3** 

Compare: 23,465 () 23,654

Since the digits in the ten-thousands place and the thousands place match, we look to the hundreds place to make the comparison.

23,465 < 23,654

# Example 4

Three of the longest underwater tunnels in North America are in New York City. The Brooklyn-Battery Tunnel is 9117 feet long, the Lincoln Tunnel is 8216 feet long, and the Holland Tunnel is 8558 feet long. Write the names and lengths of these tunnels in order from shortest to longest.

Arranging the numbers in order from least to greatest arranges the tunnels in order from shortest to longest: Lincoln Tunnel (8216 feet), Holland Tunnel (8558 feet), Brooklyn-Battery Tunnel (9117 feet).

# Ordinal Numbers

Numbers used to name position or order are called **ordinal numbers.** The following table shows two ways to write the first twelve ordinal numbers.

#### Math Language

**Cardinal numbers** such as 1, 2, 3, 4, and 5 tell *how many.* Ordinal numbers such as first, second, and third tell *which one.* 

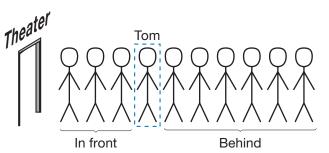
#### Ordinal Numbers for 1–12

1st	first
2nd	second
3rd	third
4th	fourth
5th	fifth
6th	sixth
7th	seventh
8th	eighth
9th	ninth
10th	tenth
11th	eleventh
12th	twelfth



# Tom was the fourth person in a line of ten people waiting for a movie. How many people were in front of Tom? How many people were behind Tom?

We draw a picture to illustrate the problem.



By counting people in our picture, we find that there are **three people in front** of Tom and **six people behind** him.

# Lesson Practice

**Represent** Use words to name each number. (*Hint:* Begin by writing the number with a comma.)

- **a.** 36420
- **b.** \$12300
- **c.** 4567

**Represent** Use digits to write each number:

- d. sixty-three thousand, one hundred seventeen
- e. two hundred fifty-six thousand, seven hundred
- f. fifty thousand, nine hundred twenty-four

- g. seven hundred fifty thousand dollars
- h. **Analyze** Christina was the sixth person in a line of ten people. How many people were in front of Christina, and how many people were behind her?

Written Practice

Distributed and Integrated

\*1. Model Use money manipulatives to answer the question in this word problem:

Nevaeh had \$462. After she was paid \$88 rent, how much money did Nevaeh have?

- 2. Which digit is in the tens place in 567?
- **3. Represent** Use digits to write seven hundred seven.
- **4.** Mount Everest, in Asia, has the highest peak in the world. The peak is <sup>(7)</sup> 29,035 feet above sea level. Use words to name this height.
- **5.** Find the sum of 54 and 246.

Find each sum:

	\$463		-	709
(6)	+ \$364	<sup>(6)</sup> + \$414	(6) +	- 314

**Predict** Find the seventh term in each counting sequence:

<b>9.</b> 10, 20, 30,	<b>10.</b> 5, 10, 15,	
<b>11.</b> 6, 12, 18,	<b>12.</b> 7, 14, 21,	
<b>13.</b> 8, 16, 24,	<b>14.</b> 9, 18, 27,	
<b>15.</b> Compare: two hundred fifty $\bigcirc$ two hundred fifteen		
<b>* 16.</b> (4, 6) <b>Explain</b> Compare. How ca	n you answer the comparison without	

365 + 366 () 365 + 365

Find each sum:

<b>17.</b> \$436		<b>19.</b> 506	
<sup>(6)</sup> \$ 72	<sup>(6)</sup> 493	<sup>(6)</sup> 79	
+ \$ 54	+ 147	+ 434	

**20. Represent** Write this comparison using digits and a comparison symbol:

Four hundred eight is less than four hundred eighty.

**21. Multiple Choice** We can count to 24 by 2s or by 3s. We do not count to 24 when counting by \_\_\_\_\_.

**A** 4s **B** 5s **C** 6s **D** 8s

**Classify** Describe each number as odd or even:

- \***22.** 1969 **\*23.** 1492 **\*24.** 1776
  - **25.** The smallest even three-digit number is 100. What is the smallest odd <sup>(2)</sup> three-digit number?
- \*26. Analyze Of the twelve people in line, Rosario was fifth. How many people were in front of Rosario? How many were behind her?
- \*27. **Predict** Is the twentieth term in this counting sequence odd or even?

1, 3, 5, 7, ...

**28.** Five birds were perched on a branch. Could half of the birds fly away? Why or why not?

**Generalize** Use this table to answer problems **29** and **30**:

Number of Dimes	1	2	3	4
Number of Pennies	10	20	30	40

**29.** Write a rule that describes how to find the number of pennies for any <sup>(1)</sup> number of dimes.

**30.** How many pennies are represented by eight dimes?

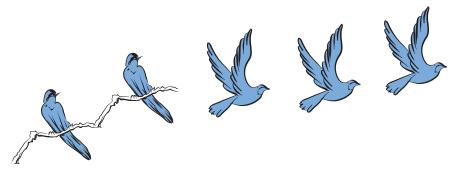


Power Up

# Relationship Between Addition and Subtraction

facts	Power Up A
count aloud	Count up and down by 50s between 0 and 500. Count up and down by 500s between 0 and 5000.
mental math	<ul> <li>a. Addition: 3000 + 3000</li> <li>b. Addition: 5000 + 5000</li> <li>c. Addition: 350 + 450</li> <li>d. Addition: 370 + 580</li> <li>e. Money: \$275 + \$25</li> <li>f. Money: \$350 + \$500</li> <li>g. Addition: 750 + 750</li> </ul>
problem solving	<b>h. Measurement:</b> 250 millimeters + 750 millimeters Choose an appropriate problem-solving strategy to solve this problem. The sum of the six numbers that have the digits 1, 2, and 3 is 1332. What is the sum of the six three-digit numbers that each have the digits 2, 4, and 6? What do you notice about the two sums?
New Concept	

Subtraction involves taking one number from another number. If five birds were perched on a branch and three flew away, then two birds would be left on the branch.



A number sentence for this problem is

5

$$5 - 3 = 2$$

We read this number sentence, "Five minus three equals two." The dash (–) between the 5 and the 3 is called a **minus sign**. The minus sign tells us to subtract the number to the right of the sign from the number to the left of the sign. Order matters when we subtract. The answer to 5 - 3 does not equal the answer to 3 - 5. When we see 5 - 3, we must start with 5 and subtract 3.

When a subtraction problem is written in a column (with one number above the other) we start with the top number and subtract the bottom number. The two forms below mean the same thing. With both forms, we start with 5 and subtract 3.

$$-3 = 2$$
 5  
 $-\frac{3}{2}$ 

The answer when we subtract is called a **difference**. We can say "the difference of 5 and 3 is 2."

#### Example 1

### When 7 is subtracted from 12, what is the difference?

We start with 12 and subtract 7. If we write the numbers horizontally, we write the 12 on the left. If we write the numbers in a column, we position the 12 on top and the 7 below the 2 in 12. This way, digits with the same place value are in the same column. We find that the difference of 12 and 7 is **5**.

$$12 - 7 = 5$$
 12  
 $-\frac{7}{5}$ 

### Example 2

### What is 8 minus 3?

The word *minus* means "take away." For this problem, we take 3 away from 8. When we see the word *minus*, we may put a minus sign in its place. We find that 8 minus 3 equals **5**.

 $8 - 3 = 5 \qquad 8 \\ - 3 \\ - 3 \\ 5 \end{cases}$ 

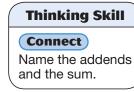
Addition and subtraction are closely related. We say that addition and subtraction are **inverse operations** because one operation "undoes" the other. If we add 3 to 5, we get 8. If we then subtract 3 from 8, we again have 5. By subtracting 3, we undid the addition of 3.

For every addition fact, we can form a subtraction fact. With the numbers 2, 3, and 5, for example, we can form two addition facts and two subtraction facts.

2	5	3	5
+ 3	- 3	+ 2	- 2
5	2	5	3

We call the three numbers 2, 3, and 5 a fact family.

**Example 3** Write two addition facts and two subtraction facts for the fact family 3, 4, and 7. 3 + 4 = 74 + 3 = 7 7 - 3 = 4 7 - 4 = 3Lesson Practice Subtract: **a.** 17 – 9 **b.** 12 – 8 **c.** 15 – 9 **e.** 17 – 8 **f.** 16 – 8 **d.** 11 – 5 Write two addition facts and two subtraction facts for each fact family: **g.** 7, 8, 15 **h.** 5, 7, 12



Written Practice

- **1.** Which digit in 3654 is in the thousands place?
- \*2. Name the five odd, one-digit numbers.
  - 3. When seven is subtracted from 15, what is the difference?
  - 4. When 56 is added to 560, what is the sum?
  - 5. What is seven minus four?
  - 6. What is sixty-four plus two hundred six?
- **\*7. Represent** Use words to name \$812,000.
- \*8. **Represent** Use digits to write eight hundred two.
  - 9. Write a two-digit odd number using 5 and 6.
- \* **10. Represent** Use words to name the number for "4 hundreds plus <sup>(3)</sup> 4 tens plus 4 ones."

Generalize Describe the rule for each counting sequence, and write the ninth term.

- **11.** 6, 12, 18, ... **12.** 3, 6, 9, ...
- **13. Connect** Write two addition facts and two subtraction facts for the fact family 4, 8, and 12.
- \*14. Verify Think of two odd numbers and add them. Is the sum odd or even? Explain how you found your answer.

Subtract to find each difference:

<b>15.</b> 18 – 9	<b>16.</b> 15 – 7	<b>17.</b> 12 – 5
<b>18.</b> 11 – 8	<b>19.</b> 14 – 6	<b>20.</b> 13 – 9

Add to find each sum:

**21.** \$36 + \$403 + \$97**22.** 572 + 386 + 38**23.** 47 + 135 + 70**24.** \$590 + \$306 + \$75

**25. Analyze** If the greatest odd number in the list below is added to the smallest even number in the list, then what is the sum?

364 287 428 273

- **26.** Write the smallest four-digit whole number. Is the number odd or even?
- 27. Half of the 18 students were girls. How many girls were there?
- **28.** From Adelio's house to school and back is five miles. How far is it from <sup>(2)</sup> Adelio's house to school?

**Generalize** Use this table to answer problems **29** and **30**:

Number of Weeks	1	2	3	4
Number of Days	7	14	21	28

**29.** Write a rule that describes how to find the number of weeks for any <sup>(1)</sup> number of days.

**30.** How many weeks are represented by fifty-six days?



The United States flag has 13 stripes. Seven stripes are red and six are white. Use this information to write a fact family that contains two addition equations and two subtraction equations.



Power Up

# Practicing the Subtraction Algorithm

facts	Power Up B
count aloud	Count up and down by 50s between 0 and 500. Count up and down by 500s between 0 and 5000.
mental	a. Money: \$250 + \$250
math	<b>b. Addition:</b> 6000 + 6000
	<b>c. Money:</b> \$75 + \$125
	<b>d. Addition:</b> 750 + 750
	e. Measurement: 60 degrees – 20 degrees
	f. Subtraction: 600 – 200
	g. Subtraction: 6000 – 2000
	h. Addition: 860 + 70
problem solving	Choose an appropriate problem-solving strategy to solve this problem. The letters P, T, and A can be arranged in six different orders. Write the six possible orders, and circle the ones that spell words.



We may find a subtraction answer by counting, by using objects, or by remembering fact families. When subtracting larger numbers, it is helpful to have a method. Recall from Lesson 6 that a method for solving a problem is an *algorithm*. In this lesson we will practice an algorithm for subtraction. We will use a money example to help us understand the algorithm. **Model** Use your \$100, \$10, and \$1 money manipulatives to model the following problem.

Maribel has \$524. She must pay Tynice \$58 for rent. After she pays Tynice, how much money will she have?

We will use five \$100 bills, two \$10 bills, and four \$1 bills to show how much money Maribel has.



From \$524, Maribel must pay Tynice \$58, which is five \$10 bills and eight \$1 bills. Maribel has enough money to pay Tynice, but she doesn't have enough \$10 bills and \$1 bills to pay her the exact amount. Before Maribel pays Tynice, she must exchange one \$10 bill for ten \$1 bills. Then she will have enough ones.



**Discuss** Maribel still does not have enough tens. What does she need to do?

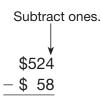


Now Maribel can pay Tynice with five \$10 bills and eight \$1 bills. Taking away 5 tens and 8 ones leaves this much:



After she pays Tynice, Maribel will have \$466.

We exchanged bills to show the subtraction. We also exchange when we use the pencil-and-paper algorithm. We write the subtraction problem and begin by subtracting the ones.



We cannot subtract \$8 from \$4. We need more ones. We look at the tens column and see 2 tens. We exchange 1 ten for 10 ones, which gives us 1 ten and 14 ones. Now we can subtract the ones.

Next we subtract the tens. We cannot subtract 5 tens from 1 ten, so we will exchange again. This time we exchange 1 hundred for 10 tens, which gives us 4 hundreds and 11 tens. Now we finish subtracting.

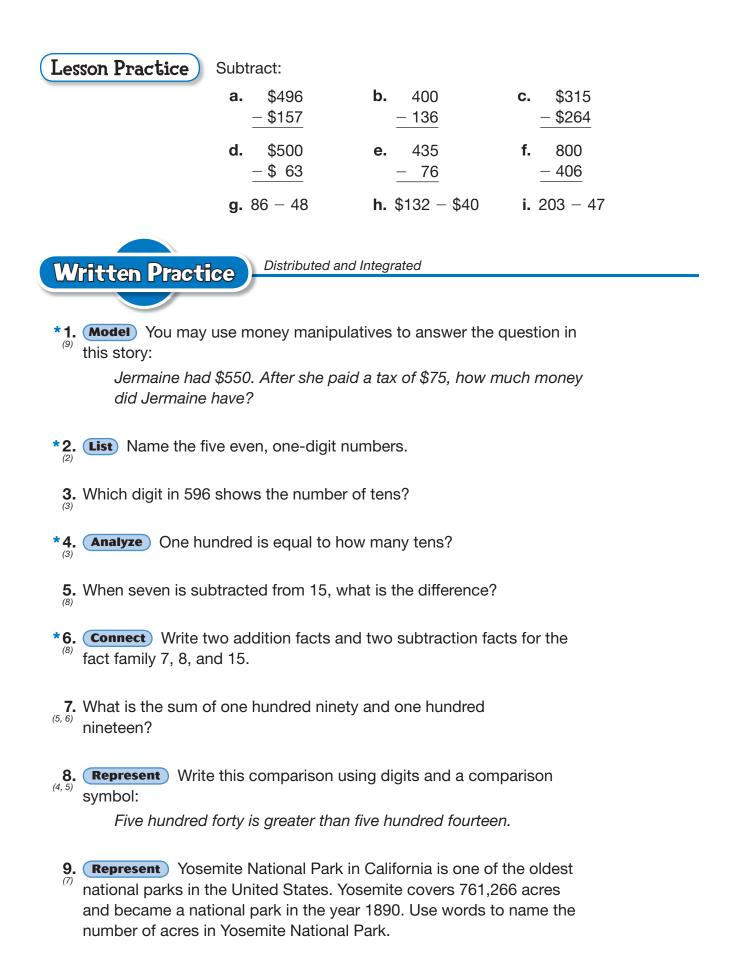
	4	<sup>1</sup> 1	
	\$5	2	4
_	\$	5	8
	\$4	6	6

**Connect** How are these exchanges similar to the exchanges using paper money?

Since the value of every column is 10 times the value of the column to its right, we can use this method any time we come to a column in which we cannot subtract.

	\$346 \$264	b. 219 – 73	c. 600 <u>– 123</u>
_	<sup>2</sup> \$3 <sup>1</sup> 46 <u>\$264</u> <b>\$82</b>	<b>b.</b> $2^{1}19$ -73 <b>146</b>	
с. 	5,9 Ø00 or <u>123</u> <b>477</b>	59600-123477	

Notice part **c.** When we try to exchange 1 ten for 10 ones, we find that there are zero tens in the tens column. We must go to the hundreds column to create some tens. We show two ways to do this. In the first method we exchange 1 hundred for 10 tens, and then we exchange 1 of those tens for 10 ones. In the second method we think of 600 as 60 tens. Taking 1 of the tens leaves 59 tens. Some people think this method of subtracting across zeros is easier and neater than the first.



**10.** Analyze Write a three-digit even number less than 200 using the digits 1, 2, and 3.

<b>11.</b> \$346	<b>12.</b> 56	<b>13.</b> \$219	<b>14.</b> 600
<sup>(9)</sup> <u>- \$178</u>	(9) <u>- 38</u>	(9) - \$ 73	(9) - 321
<b>15.</b> 300	<b>16.</b> \$500	<b>17.</b> 608	<b>18.</b> 415
(9) <u>- 124</u>	\$246	(9) <u>- 314</u>	<sup>(9)</sup> <u>- 378</u>
<b>19.</b> \$787	20.  573  90  + 438	<b>21.</b> \$645	<b>22.</b> 429
\$156		\$489	(6) 85
<u>+</u> \$324		+ \$ 65	+ 671

**Predict** Write the ninth term in each counting sequence:

- **23.** 7, 14, 21, ... **24.** 9, 18, 27, ... **25.** 8, 16, 24, ...
- **26.** (*Classify*) Is three hundred seventy an odd number or an even number? Explain how you know.
- **27.** Compare. (Try to answer the comparison before subtracting. Then subtract and compare.)

31 - 12 () 31 - 15

28. Half of 20 is 10. What number is half of 21?

**Generalize** Use this table to answer problems **29** and **30**:

Number of Insects	1	2	3	4
Number of Legs	6	12	18	24

**29.** Write a rule that describes how to find the number of legs for any <sup>(1)</sup> number of insects.

30. What number of legs represents 7 insects?

LESSON 10

# • Missing Addends

Power Up

facts	Power Up B
count aloud	Count up and down by 25s between 0 and 200. ( <i>Hint:</i> Think of quarters.) Count up and down by 20s between 0 and 200.
mental	<b>a. Money:</b> \$5000 + \$4500
math	<b>b. Subtraction:</b> 6000 - 4000
	<b>c. Money:</b> \$750 + \$250
	<b>d. Addition:</b> 380 + 90
	<b>e. Subtraction:</b> 500 - 400
	f. Measurement: 125 yards + 125 yards
	g. Addition: 640 + 260
	<b>h. Number Sense:</b> 6 + 6 - 2 + 5
problem solving	Choose an appropriate problem-solving strategy to solve this problem. Arrange the letters R, T, and A in six different orders. Circle the arrangements that spell words.
New Concept	

In the number sentence below, there is a missing addend. The letter w is used to represent the missing addend.

$$8 + w = 15$$

#### **Math Symbols**

Any uppercase or lowercase letter may be used to represent a number. A number sentence with an equal sign is often called an **equation.** Since eight plus seven equals 15, we know that the missing addend in this equation is seven. Notice that we can find a missing addend by subtracting. For the number sentence 8 + w = 15, we subtract eight from 15 to find the missing number:

$$15 - 8 = 7$$

#### Example 1

Find the missing addend:
24
$\frac{+m}{37}$
37
There are two addends and the sum.
24 addend + <u>m</u> addend 37 sum
One of the addends is 24. The sum is 37. We subtract 24 from 37 and find that the missing addend is <b>13.</b> Then we substitute 13 into the original problem to be sure the answer is correct.
$\frac{37}{-24} \xrightarrow{} \frac{24}{+13} \xrightarrow{}$
<b>Discuss</b> W/by do we use addition to check a subtraction

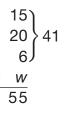
**Discuss** Why do we use addition to check a subtraction problem?

**Example 2** 

#### Find the missing addend:

#### 15 + 20 + 6 + w = 55

In this equation there are four addends and the sum. The known addends are 15, 20, and 6. Their total is 41.



So 41 plus *w* equals 55. We can find the missing addend by subtracting 41 from 55, which gives us **14.** Then we check the answer.

		15 20
	Ξ	55   6- 41- 14   + 14- 55
	We see that the answer is	correct.
Example	3	•••••••••••••••••••••••••••••••••••••••
	A baseball team has nine baseman, second basem called infielders.	e players. Four of the players (the first an, shortstop, and third baseman) are sed to find the number of players on elders?
	<b>A</b> $n + 5 = 9$	<b>B</b> $4 + n = 9$
	<b>C</b> $9 + 4 = n$	<b>D</b> $5 + 9 = n$
	( )	plus the number of other players on the se equation <b>B</b> to find the number of other
Lesson Practice	Find each missing adder	nd:
	<b>a.</b> 35 + <i>m</i> = 67	<b>b.</b> <i>n</i> + 27 = 40
	<b>c.</b> $5 + 7 + 9 + f = 30$	<b>d.</b> $15 + k + 10 + 25 = 70$
	e. <b>Explain</b> How do y reasonable?	ou know your answers are
	pocket. She gave so 6 pebbles. Select ar how many pebbles	<b>C</b>
	<b>A</b> $16 - 6 = g$	<b>B</b> $g - 16 = 6$
	<b>C</b> $16 - g = 6$	<b>D</b> $g - 6 = 16$
Written Prac	Distributed and Integ	rated
* <b>1.</b> Model Use m	noney manipulatives to answ	er the question in this word

Yvette won \$200 in an essay contest. If she had \$467 before she won the contest, how much money did she have after she won the contest?

- **2. Connect** Write two addition facts and two subtraction facts for the fact family 4, 5, and 9.
- **3. Represent** Write this comparison using digits and a comparison symbol:

Six hundred thirteen is less than six hundred thirty.

**\*4. Analyze** Use the digits 4, 5, and 6 to write a three-digit odd number that is greater than 500.

**5.** 34 + m = 61

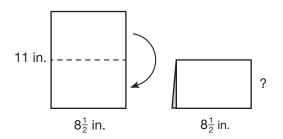
- **6.** What is five hundred ten minus fifty-one?
  - **7.** Which digit in 325,985 shows the number of hundreds?
  - **8. Multiple Choice** We can count to 30 by 3s or by 10s. We do not count to 30 when counting by
    - **A** 2s **B** 4s **C** 5s **D** 6s
  - **9.** Think of one odd number and one even number and add them. Is the <sup>(2)</sup> sum odd or even?
- **10.** (4,9) Compare. How can you answer the comparison without subtracting?

<b>11.</b> \$363	<b>12.</b> 400	<b>13.</b> \$570	<b>14.</b> 504
<sup>(9)</sup> <u>- \$179</u>	(9) - 176	<sup>(9)</sup> <u>- \$ 91</u>	<sup>(9)</sup> - 175
<b>15.</b> \$367	<b>16.</b> 179	<b>17.</b> \$305	<b>18.</b> 32
\$ 48	484	(6) \$897	(6) 248
<u>+ \$135</u>	+ 201	+ \$725	+ 165
<b>19.</b> \$463 - \$85		<b>20.</b> $432 + 84 + 578$	3
<b>21.</b> $18 + w = 42$		<b>22.</b> $12 + r = 80$	

**Conclude** Write the next four terms in each counting sequence:

**23.** 3, 6, 9, 12, ... **24.** 4, 8, 12, 16, ... **25.** 6, 12, 18, 24, ...

- **\*26.** How many \$100 bills are needed to total \$1000? (3, 7)
- \*27. Analyze Sabrina folded an  $8\frac{1}{2}$ -by-11-inch piece of paper in half as shown below. The folded paper made a rectangle that was  $8\frac{1}{2}$  inches by how many inches?



**28.** (2) **Explain** Is half of 37,295 a whole number? Why or why not?

**Generalize** Use this table to answer problems **29** and **30**:

Number of Dogs	1	2	3	4
Number of Paws	4	8	12	16

**29.** Write a rule that describes how to find the number of dogs for any  $^{(1)}$  number of paws.

**30.** How many dogs are represented by 28 paws?



Nika, Rhonda, and Alpesh collect trading cards. Together they have a total of 63 cards. If Nika has 27 cards and Rhonda has 15 cards, how many cards does Alpesh have?



# Translating and Writing Word Problems

In this investigation we will study four types of word problems: problems about **combining**, problems about **separating**, problems about **equal groups**, and problems about **comparing**.

We will see one example of each type of problem. All the problems contain three numbers. A problem becomes a word problem when one of its numbers is replaced with a question. We will make three different word problems for each problem in this investigation by replacing the numbers with questions. In later lessons we will practice solving word problems.

# Word Problems about Combining

We combine two (or more) quantities by adding them together. We start with some and add some more. Here is a problem about combining:

- a. The troop hiked 8 miles in the morning.
- **b.** The troop hiked 7 miles in the afternoon.
- c. Altogether, the troop hiked 15 miles.

Notice that there are three numbers. The numbers in **a** and **b** add up to the number in **c**. If we know any two of the numbers, we can figure out the third number. The problem is written in three sentences.

**Formulate** Suppose sentence **a** were missing. Read sentences **b** and **c** and then write a question that asks for the number in sentence **a**. Start the question with the words, "How many miles. ..."

**Formulate** Now suppose sentence **b** were missing from the problem. Read sentences **a** and **c** and then write a question that asks for the number in sentence **b**. Start with the words, "How many miles. ..."

**Formulate** Finally, suppose sentence **c** were missing. Read sentences **a** and **b** and then write a question that asks for the number in **c**. This time start the question with the words, "Altogether, how many miles. ..."

## Word Problems about Separating

We separate one quantity from a larger quantity by taking some away, or subtracting. Here is a problem about separating:

- d. Jack went to the store with \$28.
- e. Jack spent \$12 at the store.
- f. Jack left the store with \$16.

This is a problem about Jack's money. Jack had some money; then some money "went away" at the store. There are three numbers in the problem. If one of the numbers were missing, we could figure out the missing number.

**Formulate** Suppose sentence **d** were missing. Read sentences **e** and **f** and then write a question that asks for the number in sentence **d**. Start with the words, "How much money. ..."

**Formulate** Now suppose sentence **e** were missing. Read sentences **d** and **f**; and then write a question that asks for the number in sentence **e**. Start with the words, "How much money. ..."

**Formulate** Finally, suppose sentence **f** were missing. Read sentences **d** and **e** and then write a question that asks for the number in sentence **f**.

## Word Problems about Equal Groups

Some problems are about items that are clustered in groups of equal size. These problems might describe the number of groups, the number in each group, and/or the total number in all groups. By multiplying the number in each group by the number of groups, we can find the total in all groups. Here is an example of an "equal groups" problem:

At Lincoln School there are the same number of students in each fifth grade class.

- g. At Lincoln School there are 4 classes of fifth grade students.
- h. There are 30 students in each fifth grade class.
- i. Altogether, there are 120 fifth grade students at Lincoln School.

Again we see three numbers in the problem. If we know two of the numbers, we can figure out the third number.

**Formulate** Suppose sentence **g** were missing. Read sentences **h** and **i** and then write a question that asks for the number in sentence **g**. Start with the words, "How many classes. ..."

**Formulate** Now suppose sentence **h** were missing. Read sentences **g** and **i** and then write a question that asks for the number in sentence **h**. Start with the words, "How many students ..."

**Formulate** Finally, suppose sentence **i** were missing. Read sentences **g** and **h** and then write a question that asks for the number in sentence **i**. Start with the words, "Altogether, how many ..."

## Word Problems about Comparing

One way to compare two numbers is to find how much larger or how much smaller one number is than the other. By subtracting the smaller number from the larger number, we find the difference of the numbers. Consider this problem about comparing:

- j. Abe is 5 years old.
- k. Gabe is 11 years old.
- I. Gabe is 6 years older than Abe.

A comparison may be stated two ways. For example, sentence I could have been written, "Abe is 6 years younger than Gabe."

**Formulate** Once again, our problem has three numbers. If we know two of the numbers, we can figure out the third number. Suppose sentence **j** were missing. Read sentences **k** and **l** and then write a question that asks for the number in sentence **j**.

**Formulate** Now suppose sentence **k** were missing. Read sentences **j** and **l** and then write a question that asks for the number in sentence **k**.

**Formulate** Finally, suppose sentence I were missing. Read sentences j and k and then write a question that asks for the number in sentence I. You should be able to phrase the question two different ways.

# Writing Word Problems

Material needed:

• Lesson Activity 17

Use **Lesson Activity 17** to write word problems about combining, separating, multiplying, and dividing. Then illustrate one of your word problems.

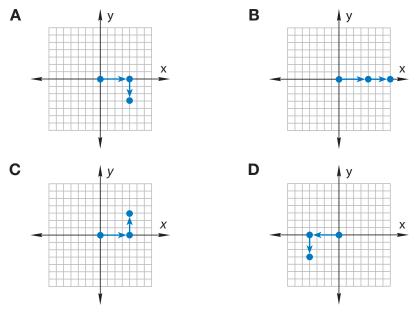
**1.** Below is a three-frame problem about Arnold's trip to the store. Help Arnold find out how much money he will get back.



- 2. Write a combining word problem that can be solved by adding.
- **3.** Write a separating word problem that can be solved by subtracting.
- **4.** Write an equal groups word problem that can be solved by multiplying.
- **5.** Write an equal groups word problem that can be solved by dividing.
- **6.** Write a comparison word problem that can be solved by subtracting.
- **7.** Select one of your problems from **2–6** and illustrate it in three frames.



a. Multiple Choice Jamaal began at his home and walked
 4 blocks east. Then he turned and walked 3 blocks north. Which diagram below best represents the path that Jamaal walked?



**b. Justify** The group of color names below were sorted by one common attribute.

Green Brown Mauve Peach Beige

These color names do not belong in the above group.

Red Lavender Blue Yellow

Name another color that belongs in the first group. Justify your answer by explaining why it is reasonable.

- **c. Multiple Choice** One hundred percent represents all of the gases in our atmosphere. About twenty-one percent of our atmosphere is oxygen. Which equation can be used to estimate the percent of our atmosphere that is *not* oxygen?
  - **A** 21% + 100% = *n*
  - **B** 100% + *n* = 21%
  - **C** 21% + n = 100%
  - **D** 21% n = 100%